REGISTRATION OF JOINT GEOMETRIC AND RADIOMETRIC IMAGE DEFORMATIONS

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ABSTRACT

We consider the problem of object registration where the observation and template differ both geometrically and radiometrically. The type of radiometric deformations we consider falls into the general framework of color constancy, and the geometric deformations are affine. Since the registration procedure employs the intensity information in both the template and observation, a solution to the color constancy problem have to be established jointly with a solution to the problem of estimating the geometric transformation.

We show that the original high-dimensional non-convex search problem, that needs to be solved in order to register the observation to the template, can be replaced by an *equivalent* problem, expressed in terms of a sequence of two *linear* systems of equations. A solution to this sequence provides an exact solution to the registration problem.

KEY WORDS

Computer Vision, Image Registration, Multidimensional Signal Processing, Nonlinear Parameter Estimation.

1 INTRODUCTION

This paper is concerned with the problem of object registration when the observation and template differ both geometrically and radiometrically. The type of radiometric deformations we consider can be classified as a part of the general framework known as *color constancy*, [1]. Thus, the problem being addressed is that of joint estimation of the radiometric and geometric deformations relating the template and observation (or in general, any two observations on the same object). More specifically, in many applications the template and observation are recorded under different illumination conditions and by different imaging devices, thus, measured intensities may vary significantly from one observation to the other. Nevertheless, in image registration one attempts to match every point in the observation to its corresponding point in the template image. The registration procedure employs the intensity information in both the template and observation, thus, a solution to the color constancy problem have to be established jointly with estimating the geometric transformation in order to achieve the desired matching in the presence of intensity variations.

An important case where type of radiometric deformations we consider appears is that of single-modal registration, where the measured intensities may non-linearly vary either due to variations in the acquisition system or due the use of different acquisition systems. Such non-linearity is typically introduced by an image acquisition system as the overall non-linearity of its various components (CCD/CMOS, amplifiers, etc.), [2]. The solution we propose in the sequel allows one to solve the registration problem, while being invariant to the effects of the non-linear response function of the camera. Thus, no *a-priori* calibration for the camera response function is required, and in fact, the need for knowledge of the camera response function is eliminated.

We begin by setting the mathematical framework required for describing and rigorously defining the registration problem. We aim at a formulation that is both inherent to some physical reality and mathematically accessible. Let S be a set (a function space in our case) and let G_L and G_R be groups that act as left-hand and right-hand transformation groups on S, respectively. Let $G = G_L \times G_R$ be the cross product group of G_L and G_R. As such, the actions of G_L and G_R on S commute, and the action of G on S is a well defined mapping $G \times S \to S$ given by $((\phi, \varphi), s) \mapsto \phi \circ s \circ \varphi$ for every $(\phi, \varphi) \in G$ and every $s \in S$. Denote by Gs the orbit of $s \in S$ under the action of G, *i.e.*, $Gs = \{\phi \circ s \circ \varphi : (\phi, \varphi) \in G\}$. Since G is a group, the set of orbits of S under the action of G forms a partition of S. The induced equivalence relation naturally introduces a notion of similarity: $s_1, s_2 \in S$ are *similar* if they belong to the same orbit, that is $Gs_1 = Gs_2$.

Now, let $S = \{f : X \to Y\}$ be a function space, and let G_L and G_R be C^1- diffeomorphism groups of Y and X respectively (*i.e.*, each element of the group is a continuously differentiable function with a continuously differentiable inverse). Consider S as a signal space (image, audio, etc.) then the action of G_R on S, that is, the right-hand composition with φ, can be thought of as a spatial/time deformation (*i.e.*, a deformation of the coordinate system) while the action of G_L on S, that is, the left-hand composition with ϕ, can be thought of as a memoryless non-linear input/output system applied to the amplitude of the signal. Hence, in image formation terminology, the physical model corresponding to the action of the (product) group G on a function space S is that of a simultaneous deformation of both geometry and radiometry. From this point of view and in the absence of noise, given two functions $f, g \in S$, the registration problem is to find (if possible) an element $(\phi, \varphi) \in G$ that makes f and g identical in the sense that $f = \phi \circ g \circ \varphi$.

Once G, the set of allowable variations, has been defined, a straightforward approach for solving the registration problem is to apply each of the elements of G to the template until a match is found. By the modeling assumption, in the absence of noise, such element is guaranteed to exist. However, even in simple cases, the number of such possible elements is infinite. This typically leads to a high-dimensional non-convex search problem, [3, 4, 5], hence, such direct approach is computationally demanding.

In this paper we elaborate on a special case of the above general problem, and consider the case where the geometric deformation is affine, that is G_R is the group of affine transformations of R^n. The radiometric deformation is due to G_L which is a C^1-diffeomorphism group of R. We show that by exploiting the algebraic structure we have just defined, the original high-dimensional non-convex search problem, that needs to be solved in order to register the observation to the template, is mapped to an *equivalent* problem, expressed in terms of a sequence of two *linear* problems. Registration is obtained by solving the two systems of linear equations.

2 PROBLEM DEFINITION

Let $\mathbf{A} \in GL_n(\mathrm{R})$ and $\mathbf{c} \in \mathrm{R}^n$ denote the parameters of the geometric coordinate transformation. Let $Q : \mathrm{R} \to \mathrm{R}$ be a strictly increasing, continuously differentiable function with a continuously differentiable inverse, representing the radiometric deformation. Let us further assume that $Q(0) = 0$. Given non-negative, bounded, Lebesgue measurable, compactly supported functions with no affine symmetry $f, g \in M_{Aff}(\mathrm{R}^n, \mathrm{R})$ (see [6] for a rigorous definition) such that

$$f(\mathbf{x}) = Q(g(\mathbf{A}\mathbf{x} + \mathbf{c})) \qquad \mathbf{x} \in \mathrm{R}^n \tag{1}$$

the problem is to *find the matrix* $\mathbf{A}$*, the translation vector* $\mathbf{c}$ *and the left-hand composition* Q.

3 AN ALGORITHMIC SOLUTION

In this section, we show that the problem can be solved in two stages: first we use an operator which is affine-invariant up to a scaling constant to isolate and estimate the left-hand composition Q; next, the solution for the function Q is used to reduce the original problem into a simpler one, where the parameters of the affine transformation, $\mathbf{A}$ and $\mathbf{c}$, are evaluated.

To simplify notations, let $\varphi_{(\mathbf{A},\mathbf{c})} : \mathrm{R}^n \to \mathrm{R}^n$ denote the affine transformation corresponding to $\mathbf{A} \in GL(\mathrm{R}, n)$ and $\mathbf{c} \in \mathrm{R}^n$, that is, $\varphi_{(\mathbf{A},\mathbf{c})}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{c}$. Using this notation (1) can be compactly rewritten as

$$f = Q \circ g \circ \varphi_{(\mathbf{A},\mathbf{c})} \tag{2}$$

Let μ denote the Lebesgue measure on the n-dimensional Euclidean space R^n and let $M(\mathrm{R}^n, \mathrm{R})$ denote the space of non-negative, bounded, Lebesgue measurable, compactly supported functions from R^n to R. Define the "empirical distribution" operator on $M(\mathrm{R}^n, \mathrm{R})$ by

$$[Th](t) = \mu\{\mathbf{x} \in \mathrm{R}^n : 0 < h(\mathbf{x}) \le t\} \tag{3}$$

for $h \in M(\mathrm{R}^n, \mathrm{R})$. Notice the following properties of T:

Lemma 1 *For a given* $h \in M(\mathrm{R}^n, \mathrm{R})$ *the following holds:*

(a) $[Th](t)$ *is a distribution function. Furthermore,*

  i. $[Th](t) = 0$ *for all* $t < 0$.

  ii. $[Th](t) = \mu\{\mathrm{supp}\{h\}\}$ *for all* $t > \sup_x h(x)$.

(b) $T\left(h \circ \varphi_{(\mathbf{A},\mathbf{c})}\right) = \left|\mathbf{A}^{-1}\right| \cdot Th$ *where* $\left|\mathbf{A}^{-1}\right| = \det\left(\mathbf{A}^{-1}\right)$.

(c) $T(W \circ h) = [Th] \circ W^{-1}$ *for any strictly increasing continuous function* $W : \mathrm{R} \to \mathrm{R}$ *such that* $W(0) = 0$.

Proof: Showing (a) is trivial. Using the definition of T and the properties of the Lebesgue measure, for all t we have

$$\begin{aligned}\left[T\left(h \circ \varphi_{(\mathbf{A},\mathbf{c})}\right)\right](t) &= \mu\left\{\mathbf{x} : 0 < \left(h \circ \varphi_{(\mathbf{A},\mathbf{c})}\right)(\mathbf{x}) \le t\right\} \\ &= \mu\{\mathbf{x} : 0 < h(\mathbf{A}\mathbf{x} + \mathbf{c}) \le t\} \\ &= \mu\left\{\mathbf{A}^{-1}\mathbf{x} - \mathbf{A}^{-1}\mathbf{c} : 0 < h(\mathbf{x}) \le t\right\} \\ &= \left|\mathbf{A}^{-1}\right| \cdot \mu\{\mathbf{x} : 0 < h(\mathbf{x}) \le t\} \\ &= \left|\mathbf{A}^{-1}\right| \cdot [Th](t)\end{aligned}$$

thus (b) is proven. By the definition of T, for all t

$$\begin{aligned}[T(W \circ h)](t) &= \mu\{\mathbf{x} \in \mathrm{R}^n : 0 < W(h(\mathbf{x})) \le t\} \\ &= \mu\left\{\mathbf{x} \in \mathrm{R}^n : 0 = W^{-1}(0) < h(\mathbf{x}) \le W^{-1}(t)\right\} \\ &= [Th]\left(W^{-1}(t)\right)\end{aligned}$$

thus (c) is proven. ∎

Applying T to relation (2), and using the above properties we obtain the following functional relation

$$\begin{aligned}Tf &= T\left(Q \circ g \circ \varphi_{(\mathbf{A},\mathbf{c})}\right) \\ &= \left|\mathbf{A}^{-1}\right| \cdot T(Q \circ g) = \left|\mathbf{A}^{-1}\right| \cdot [Tg] \circ Q^{-1}\end{aligned} \tag{4}$$

Let $F(t) = [Tf](t)$ and $G(t) = [Tg](t)$. Then, for all t we have the following relation

$$F(t) = \left|\mathbf{A}^{-1}\right| \cdot G\left(Q^{-1}(t)\right) \tag{5}$$

The geometric deformation is affine, thus $\left|\mathbf{A}^{-1}\right|$, the Jacobian of $\varphi_{(\mathbf{A},\mathbf{c})}$, can be easily obtained, for example by

$\left|\mathbf{A}^{-1}\right| = \frac{\mu\{\text{supp}\{f\}\}}{\mu\{\text{supp}\{g\}\}}$. We thus conclude that F and G are functionally related through a right-hand composition Q^{-1}, (*i.e.*, a diffeomorphic deformation of coordinates) and a constant gain that depends only on the Jacobian of $\varphi_{(\mathbf{A},\mathbf{c})}$. In other words, using the operator T we have "converted" a functional relation expressed by a left-hand composition (*i.e.*, "radiometric deformation") into a new functional relation expressed by a right-hand composition (*i.e.*, "geometric deformation").

This new estimation problem is of the form considered in [7]. However, since F and G are distribution functions, they are not compactly supported. Further "compactification" is required in order to allow one to employ the method proposed in [7] for the estimation of C^1-diffeomorphic deformations of compactly supported signals. Thus, let $\varepsilon > 0$ be some arbitrarily small number and let

$$\omega_\varepsilon(t) = \begin{cases} t & , t \le \sup_\tau F(\tau) - \varepsilon \\ 0 & , elsewhere \end{cases} \tag{6}$$

Next, let $\tilde{F} = \omega_\varepsilon \circ F$ and $\tilde{G} = \omega_\varepsilon \circ \left(\left|\mathbf{A}^{-1}\right| \cdot G\right)$. By left composing ω_ε on both sides of (5), for all t we have

$$\tilde{F}(t) = \tilde{G}\left(Q^{-1}(t)\right) \tag{7}$$

Thus, $\tilde{F}$ and $\tilde{G}$ are non-negative, bounded, compactly supported, Lebesgue measurable functions from R to itself, related by the right-hand composition Q^{-1}, which by assumption is a C^1-diffeomorphism.

Let $\{e_i\}$ be a countable basis of $L_2(\text{supp}\{\tilde{F}\})$. By assumption Q' is continuous, thus, it is in $L_2(\text{supp}\{\tilde{F}\})$ and can be represented as

$$Q'(t) = \sum_i b_i e_i(t) \tag{8}$$

Using the estimation algorithm proposed in [7], any finite order model of the type (8) can now be solved for the coefficients $\{b_i\}$ by means of solving a system of linear equations. Since $Q(0) = 0$, Q can be easily obtained by integration, which completes the estimation of the mapping Q.

Having estimated Q, we can now estimate the geometric deformation: Notice that (2) can be written as

$$f = [Q \circ g] \circ \varphi_{(\mathbf{A},\mathbf{c})} = \tilde{g} \circ \varphi_{(\mathbf{A},\mathbf{c})} \tag{9}$$

where we define $\tilde{g} = Q \circ g$. Since Q has been estimated and g is known, $\tilde{g}$ represents a "new" template. Thus, (9) describes the relation of two known functions $f, \tilde{g} \in M_{Aff}(\mathrm{R}^n, \mathrm{R})$ related by an affine transformation of the coordinates. This equivalent problem can be easily solved for the unknown $\mathbf{A}$ and $\mathbf{c}$ using the algorithms proposed in [6, 8], again by means of solving a low-dimensional system of linear equations.

Based on the conclusions in [6, 7], we conclude that if the derivative of Q admits a *finite* order representation in (8), and in the absence of noise, the overall solution for both the affine deformation and the radiometric distortion is completely determined and *exact*.

4 NUMERICAL EXAMPLE

The following example illustrates the operation of the proposed algorithm on a two-dimensional RGB image of a parrot shown at the top of Figure 2. The template image $g(x, y)$ is of dimension 1024×768 and each channel values are in the range of $[0, 1]$. The observed deformed image $f(x, y)$ is shown in the middle of Figure 2. It is a version of the template subject to both geometric and radiometric deformation. The geometric deformation is an affine transformation of coordinates (without translation). The radiometric deformation is due to three different point-wise nonlinear mappings applied to the amplitudes (intensities) of each of the template's channels. The mappings Q_R, Q_G and Q_B, respectively corresponding to the RGB channels of the image, were chosen to be polynomial, and are given by

$$Q_R(t) = t; Q_G(t) = 1.6t^3 - 0.8t^5; Q_B(t) = 2t - t^2$$

Q_R, Q_G and Q_B were individually estimated by applying the proposed algorithm on each of the image channels. The intensity mapping functions and their corresponding estimates, as obtained by the proposed algorithm, are shown in Figure 1. The estimation errors (MSE) are $4.7980 \cdot 10^{-9}$; $1.4239 \cdot 10^{-7}$ and $1.3127 \cdot 10^{-7}$, respectively.

The affine deformation of coordinates is given by

$$\mathbf{A} = \begin{bmatrix} 1.3 & 0.3 \\ 0.1 & -1.3 \end{bmatrix}$$

The estimate is obtained jointly for all channels by stacking the sets of equations obtained using the individual channel information (see [6, 8]) into a single over–determined system. This system is solved for the elements of A using a least squares solution. This estimate yields

$$\hat{\mathbf{A}} = \begin{bmatrix} 1.3016 & 0.3008 \\ 0.1076 & -1.2984 \end{bmatrix}$$

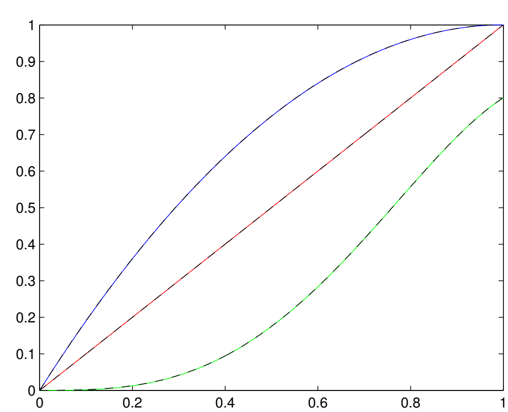

Figure 1. The radiometric deformations Q_R, Q_G and Q_B of the three image color channels (red, green and blue solid lines) and their estimates (dashed lines)

Finally, the estimated geometric deformation and the estimated amplitude mappings are applied to the original template in order to obtain an estimate of the deformed image as shown at the bottom of Figure 2.

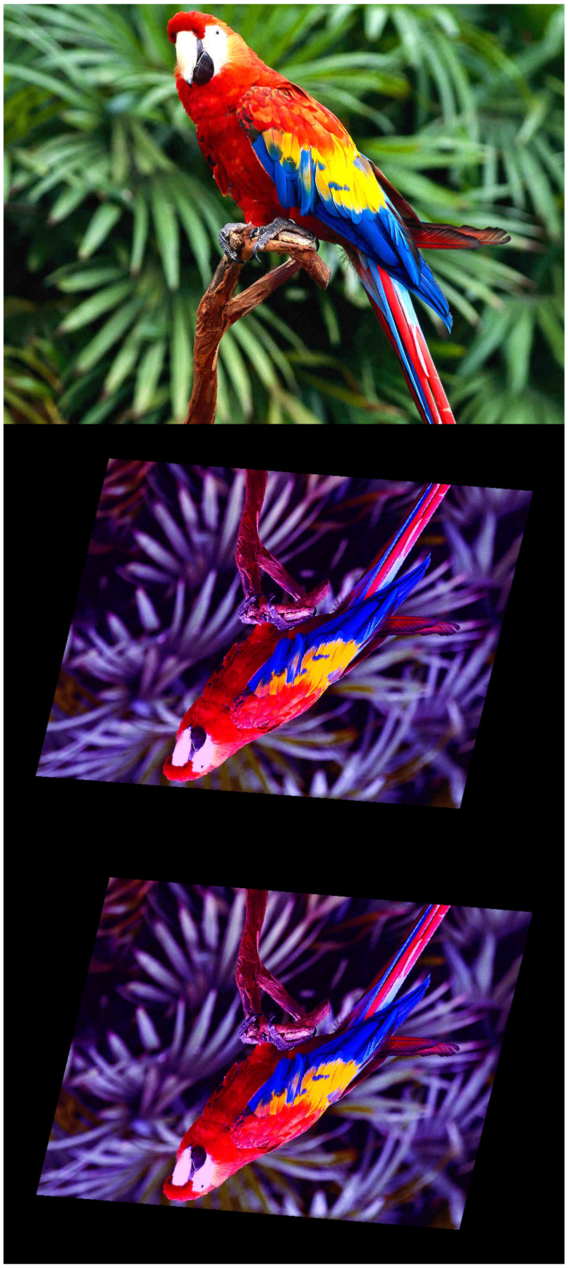

Figure 2. From top to bottom: Original template; Observation on the deformed image; Estimated deformed image obtained by applying the estimated geometric and radiometric deformations to the template.

5 CONCLUSIONS

In this paper we have addressed the question of parametric object registration, where the observation and template differ both geometrically and radiometrically. In particular we elaborate on a special case of the above general problem, where the geometric deformation is affine, and the radiometric deformation is continuously differentiable with a continuously differentiable inverse. We show that by exploiting the algebraic structure defined in the paper, the original high-dimensional non-convex search problem, that has to be solved in order to register the observation to the template, is mapped to an *equivalent* problem, expressed in terms of a sequence of two *linear* problems. Exact registration, regardless of the magnitude of the deformations, is obtained by solving the two systems of linear equations.

The principles of the proposed method and the general framework into which the problem of joint estimation of geometric and radiometric deformations has been casted to, may be further exploited to address generalizations of the model described in this paper. Among the extensions being considered are: noise analysis, analysis in the case where the geometric deformation is elastic, and non-invertible intensity mappings.

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